1. A dentist knows from past records that 10% of customers arrive late for their appointment.

A new manager believes that there has been a change in the proportion of customers who arrive late for their appointment. two tailed

A random sample of 50 of the dentist's customers is taken.

(a) Write down

- a null hypothesis corresponding to no change in the proportion of customers who arrive late
- an alternative hypothesis corresponding to the manager's belief

(1)

(b) Using a 5% level of significance, find the critical region for a two-tailed test of the null hypothesis in (a)
You should state the probability of rejection in each tail, which should be less than 0.025

(3)

(c) Find the actual level of significance of the test based on your critical region from part (b)

(1)

The manager observes that 15 of the 50 customers arrived late for their appointment.

(d) With reference to part (b), comment on the manager's belief.

(1)

a) $H_0: p = 0.1$ $\quad H_1: p \neq 0.1$ ①

b) Let X be the number of late customers.
Assume H_0 correct: $X \sim B(50, 0.1)$ ①

$P(X \leq 1) = 0.0338 > 0.025$
$P(X \leq 0) = 0.00515 < 0.025$ ←

$P(X \geq 9) = 1 - P(X \leq 8) = 0.0579 > 0.025$
$P(X \geq 10) = 1 - P(X \leq 9) = 0.0245 < 0.025$ ←

CR: $X = 0$ and $X \geq 10$ ①

$P(X = 0) = 0.00515$ $\quad P(X \geq 10) = 0.0245$ ①

c) level of significance $= 0.00515 + 0.0245$
$= 0.0297$ (3sf) 1

d) 15 is in the critical region, so there is evidence to support the manager's belief. 1

2. A machine fills packets with sweets and $\frac{1}{7}$ of the packets also contain a prize.

The packets of sweets are placed in boxes before being delivered to shops.
There are 40 packets of sweets in each box.

The random variable T represents the number of packets of sweets that contain a prize in each box.

(a) State a condition needed for T to be modelled by $B(40, \frac{1}{7})$

(1)

A box is selected at random.

(b) Using $T \sim B(40, \frac{1}{7})$ find

(i) the probability that the box has exactly 6 packets containing a prize,

(ii) the probability that the box has fewer than 3 packets containing a prize.

(2)

Kamil's sweet shop buys 5 boxes of these sweets.

(c) Find the probability that exactly 2 of these 5 boxes have fewer than 3 packets containing a prize.

(2)

Kamil claims that the proportion of packets containing a prize is less than $\frac{1}{7}$

A random sample of 110 packets is taken and 9 packets contain a prize.

(d) Use a suitable test to assess Kamil's claim.
You should

- state your hypotheses clearly
- use a 5% level of significance

(4)

a) The probability of a packet containing a prize is constant. 1

b) $T \sim B(40, \frac{1}{7})$

(i) $P(T = 6) = 0.1727\ldots = 0.173$ (3 s.f.) 1

(ii) $P(T < 3) = P(T \leq 2)$

$= 0.061587\ldots = 0.0616$ (3 s.f.) 1

(c) Let r.v. K = number of boxes with fewer than 3 packets containing a prize.

$K \sim B(5, 0.0615...)$ 1

$\therefore P(K=2) = 0.031344... = 0.0313$ (3 s.f.) 1

d) Let r.v. X = number of packets containing a prize.

$X \sim B = (110, \frac{1}{7})$ 1

$H_0 : p = \frac{1}{7}$, $H_1 = p < \frac{1}{7}$ 1

$P(X \leq 9) = 0.038292...$ (which is < 0.05) 1

$\therefore$ reject H_0 since there is evidence to support Kamil's claim. 1

3. The proportion of left-handed adults in a country is 10%
Freya believes that the proportion of left-handed adults under the age of 25 in this country is different from 10%
She takes a random sample of 40 adults under the age of 25 from this country to investigate her belief.

(a) Find the critical region for a suitable test to assess Freya's belief.
You should
- state your hypotheses clearly
- use a 5% level of significance
- state the probability of rejection in each tail

(4)

(b) Write down the actual significance level of your test in part (a)

(1)

In Freya's sample 7 adults were left-handed.

(c) With reference to your answer in part (a) comment on Freya's belief.

(1)

a) $H_0: p = 0.1 \quad H_1: p \neq 0.1$ (1)

two tailed ∴ significance level = 2.5% = 0.025

Assume H_0 is correct. Let X be the number of left-handed adults under the age of 25.

$X \sim B(40, 0.1)$

$P(X = 0) = 0.0148 < 0.025$ (1)
$P(X \leq 1) = 0.0805 > 0.025$

$P(X \geq 8) = 1 - P(X \leq 7) = 0.0419 > 0.025$
$P(X \geq 9) = 1 - P(X \leq 8) = 0.0155 < 0.025$ (1)

$CR = \{X = 0\} \cup \{X \geq 9\}$ (1)

b) $0.0148 + 0.0155 = 0.0303$ (1)

c) 7 is not in the critical region, so there is insufficient evidence to support Freya's belief. (1)